Maksimov V.P. ON CONTROL PROBLEM FOR LINEAR SYSTEM UNDER IMPULSE DISTURBANCES

A control problem for linear functional differential system with time delay of the general form is considered. The purpose of controlling is prescribed with use of a finite set of linear functionals. The system is acting under impulse disturbances which result in trajectory jumps with unknown previously instants of time and values. A construction of control actions that contains both program and jumps-positional components is proposed.

Key words: functional differential systems; control problems; impulse disturbances.

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EXACT TRIANGULAR DECOMPOSITION

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Ключевые слова: matrix triangular decomposition; algorithm in commutative domain.

The main result which presented in this talk is existence in commutative domain R of the matrix triangular decomposition, which has the form: $A=PLDUQ$, where P and Q are permutation matrices, L and PLP^T are lower triangular matrices over R, U and Q^TUQ are upper triangular matrices over R, $D=\mathrm{diag}(d_1^{-1}, d_2^{-1}, .., d_r^{-1}, 0, .., 0)$ is a diagonal matrix of rank r, $d_i \in R\backslash\{0\}$, $i=1,\ldots,r$.

A matrix decomposition of a form

$$A = VwU \tag{1}$$

is called the Bruhat decomposition of the matrix A, if V and U are nonsingular upper triangular matrices and w is a matrix of permutation.

Let R be a commutative domain, F be the field of fractions over R. We want to obtain a decomposition of matrix A over domain R in the form $A=VwU$, where V and U are upper triangular matrices over R and w is a matrix of permutation, which is multiplied by some diagonal matrix in the field of fractions F. Moreover each nonzero element of w has the form $(a^i a^{i-1})^{-1}$, where a^i is some minor of order i of matrix A.

We call such triangular decomposition the Bruhat decomposition in the commutative domain R.

Let R be a commutative domain, $A=(a_{i,j})\in R^{n\times n}$ be a matrix of order n, $\alpha^k_{i,j}$ be $k\times k$ minor of matrix A which disposed in the rows $1,2,\ldots,k-1,i$ and columns $1,2,\ldots,k-1,j$ for all integers $i,j,k\in\{1,\ldots,n\}$. We suppose that the row i of the matrix A is situated at the last row of the minor, and the column j of the matrix A is situated at the last column of the minor. We denote $\alpha^0=1$ and $\alpha^k=\alpha^k_{k,k}$ for all diagonal minors ($1\leqslant k\leqslant n$). And we use the notation δ_{ij} for Kronecker delta.

Let k and s be integers in the interval $0\leqslant k<s\leqslant n$, $\mathcal{A}^k_s=(\alpha^{k+1}_{i,j})$ be the matrix of minors with size $(s-k)\times(s-k)$ which has elements $\alpha^{k+1}_{i,j}$, $i,j=k+1,\ldots,s-1,s$, and $\mathcal{A}^0_n=(\alpha^1_{i,j})= =A$.

Т h e o r e m 1 [LDU decomposition of the minors matrix].

Let $A=(a_{i,j})\in R^{n\times n}$ *be the matrix of rank* r, $\alpha^i\neq 0$ *for* $i=k,k+1,\ldots,r$, $r\leqslant s\leqslant n$, *then the matrix of minors* $\mathcal{A}_s^k$ *is equal to the following product of three matrices:*

$$\mathcal{A}_s^k = L_s^k D_s^k U_s^k = (a_{i,j}^j)(\delta_{ij}\alpha^k(\alpha^{i-1}\alpha^i)^{-1})(a_{i,j}^i). \quad (2)$$

The matrix $L_s^k=(a_{i,j}^j)$, $i=k+1\ldots s$, $j=k+1\ldots r$, *is a low triangular matrix of size* $(s-k)\times(r-k)$, *the matrix* $U_s^k=(a_{i,j}^i)$, $i=k+1\ldots r$, $j=k+1\ldots s$, *is an upper triangular matrix of size* $(r-k)\times(s-k)$ *and* $D_s^k=(\delta_{ij}\alpha^k(\alpha^{i-1}\alpha^i)^{-1})$, $i=k+1\ldots r$, $j=k+1\ldots r$, *is a diagonal matrix of size* $(r-k)\times(r-k)$.

C o n s e q u e n c e 1 [LDU decomposition of matrix A].
Let $A=(a_{i,j})\in R^{n\times n}$, *be the matrix of rank* r, $r\leqslant n$, $\alpha^i\neq 0$ *for* $i=1,2,\ldots,r$, *then matrix* A *is equal to the following product of three matrices:*

$$A = L_n^0 D_n^0 U_n^0 = (a_{i,j}^j)(\delta_{ij}(\alpha^{i-1}\alpha^i)^{-1})(a_{i,j}^i). \quad (3)$$

The matrix $L_n^0=(a_{i,j}^j)$, $i=1\ldots n$, $j=1\ldots r$, *is a low triangular matrix of size* $n\times r$, *the matrix* $U_n^0=(a_{i,j}^i)$, $i=1\ldots r$, $j=1\ldots n$, *is an upper triangular matrix of size* $r\times n$ *and* $D_n^0=(\delta_{ij}(\alpha^{i-1}\alpha^i)^{-1})$, $i=1\ldots r$, $j=1\ldots r$, *is a diagonal matrix of size* $r\times r$.

Let I_n be the identity matrix and P_n be the matrix with second unit diagonal.

C o n s e q u e n c e 2 [Bruhat decomposition of matrix A].
Let matrix $A=(a_{i,j})$ *have the rank* r, $r\leqslant n$, *and* $B=P_nA$. *Let* $B=LDU$ *be the LDU-decomposition of matrix* B. *Then* $V=P_nLP_r$ *and* U *are upper triangular matrices of size* $n\times r$ *and* $r\times n$ *correspondingly and*

$$A = V(P_rD)U \quad (4)$$

is the Bruhat decomposition of matrix A.

We are interested in the block form of decomposition algorithms for LDU and Bruhat decompositions. Let us use some block matrix notations.

For any matrix A (or A_q^p) we denote by $A_{j_1,j_2}^{i_1,i_2}$ (or $A_{q;j_1,j_2}^{p;i_1,i_2}$) the block which stands at the intersection of rows $i_1+1,\ldots,i_2$ and columns $j_1+1,\ldots,j_2$ of the matrix. We denote by $A_{i_2}^{i_1}$ the diagonal block $A_{i_1,i_2}^{i_1,i_2}$.

LDU algorithm.

Input: $(\mathcal{A}_n^k,\alpha^k)$, $0\leqslant k<n$.
Output: $\{L_n^k,\{\alpha^{k+1},\alpha^{k+2},\ldots,\alpha^n\},U_n^k,M_n^k,W_n^k\}$,
where $D_n^k=\alpha^k\mathrm{diag}\{\alpha^k\alpha^{k+1},\ldots,\alpha^{n-1}\alpha^n\}^{-1}$, $M_n^k=\alpha^k(L_n^kD_n^k)^{-1}$, $W_n^k=\alpha^k(D_n^kU_n^k)^{-1}$.

1. If $k=n-1$, $\mathcal{A}_n^{n-1}=(a^n)$ is a matrix of the first order, then we obtain

$$\{a^n,\{a^n\},a^n,a^{n-1},a^{n-1}\},\quad D_n^{n-1}=(\alpha^n)^{-1}.$$

2. If $k=n-2$, $\mathcal{A}_n^{n-2}=\begin{pmatrix}\alpha^{n-1} & \beta\\ \gamma & \delta\end{pmatrix}$ is a matrix of second order, then we obtain

$$\left\{\begin{pmatrix}\alpha^{n-1} & 0\\ \gamma & \alpha^n\end{pmatrix},\{\alpha^{n-1},\alpha^n\},\begin{pmatrix}\alpha^{n-1} & \beta\\ 0 & \alpha^n\end{pmatrix},\begin{pmatrix}\alpha^{n-2} & 0\\ -\gamma & \alpha^{n-1}\end{pmatrix},\begin{pmatrix}\alpha^{n-2} & -\beta\\ 0 & \alpha^{n-1}\end{pmatrix}\right\} \quad (5)$$

where $\alpha^n=(\alpha^{n-2})^{-1}\begin{vmatrix}\alpha^{n-1} & \beta\\ \gamma & \delta\end{vmatrix}$, $D_n^{n-2}=\alpha^{n-2}\mathrm{diag}\{\alpha^{n-2}\alpha^{n-1},\alpha^{n-1}\alpha^n\}^{-1}$.

3. If the order of the matrix $\mathcal{A}_n^k$ more than two ($0\leqslant k<n-2$), then we choose an integer s in the interval $(k<s<n)$ and divide the matrix into blocks

$$\mathcal{A}_n^k = \begin{pmatrix}\mathcal{A}_s^k & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{pmatrix}. \quad (6)$$

3.1. Recursive step

$$\{L_s^k, \{\alpha^{k+1}, \alpha^{k+2}, \ldots, \alpha^s\}, U_s^k, M_s^k,\ W_s^k\} = \mathbf{LDU}(\mathcal{A}_s^k, \alpha^k)$$

3.2. We compute

$$\widetilde{U} = (\alpha^k)^{-1} M_s^k \mathbf{B}, \quad \widetilde{L} = (\alpha^k)^{-1} \mathbf{C} W_s^k, \tag{7}$$

$$\mathcal{A}_n^s = (\alpha^k)^{-1} \alpha^s (\mathbf{D} - \widetilde{L} D_s^k \widetilde{U}). \tag{8}$$

3.3. Recursive step

$$\{L_n^s, \{\alpha^{s+1}, \alpha^{s+2}, \ldots, \alpha^n\}, U_n^s, M_n^s,\ W_n^s\} = \mathbf{LDU}(\mathcal{A}_n^s, \alpha^s)$$

3.4 *Result:*

$$\{L_n^k, \{\alpha^{k+1}, \alpha^{k+2}, \ldots, \alpha^n\}, U_n^k, M_n^k, W_n^k\},$$

where

$$L_n^k = \begin{pmatrix} L_s^k & 0 \\ \widetilde{L} & L_n^s \end{pmatrix}, \ U_n^k = \begin{pmatrix} U_s^k & \widetilde{U} \\ 0 & U_n^s \end{pmatrix}, \tag{9}$$

$$M_n^k = \begin{pmatrix} M_s^k & 0 \\ -M_n^s \widetilde{L} D_s^k M_s^k / \alpha^k & M_n^s \end{pmatrix}, \tag{10}$$

$$W_n^k = \begin{pmatrix} W_s^k & -W_s^k D_s^k \widetilde{U} W_n^s / \alpha^k \\ 0 & W_n^s \end{pmatrix}. \tag{11}$$

Complexity.

Let γ and β be constants, $3 \geqslant \beta > 2$, and let $M(n) = \gamma n^\beta + o(n^\beta)$ be the number of multiplication operations in one $n \times n$ matrix multiplication.

T h e o r e m 2.

The complexity of the decomposition is $\sim \frac{7\gamma n^\beta}{2^\beta - 2}$

The decomposition of the second order matrix needs in 7 multiplicative operations, so:

$$t(n) = 2t(n/2) + 7M(n/2), \quad t(2) = 7.$$

After summation from $n = 2^k$ to 2^1 we obtain

$$7\gamma(2^0 2^{\beta\cdot(k-1)} + \ldots + 2^{k-2} 2^{\beta\cdot 1}) + 2^{k-2} 7 = 7\gamma \frac{n^\beta - n 2^{\beta-1}}{2^\beta - 2} + \frac{7}{4} n.$$

The exact triangular decomposition.

D e f i n i t i o n 1. *A decomposition of the matrix* A *of rank* r *over a commutative domain* R *in the product of five matrices*

$$A = PLDUQ \tag{12}$$

is called exact triangular decomposition if

P *and* Q *are permutation matrces,*

L *and* PLP^T *are nonsingular lower triangular matrices over* R,

U *and* $Q^T UQ$ *are nonsingular upper triangular matrices over* R,

$D = \mathrm{diag}(d_1^{-1}, d_2^{-1}, .., d_r^{-1}, 0, .., 0)$ *is a diagonal matrix of rank* r, $d_i \in R\backslash\{0\}$, $i = 1, ..r$.

T h e o r e m 3 [Main theorem].

Any matrix over a commutative domain has an exact triangular decomposition.

Proof of this theorem you can find in [4].

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Малашонок Г.И. ТОЧНОЕ ТРЕУГОЛЬНОЕ РАЗЛОЖЕНИЕ

Основным результатом, который представлен в этой работе, является существование в коммутативной области R матрицы треугольного разложения, которая имеет вид: $A = PLDUQ$, где P и Q – перестановочные матрицы, L и PLP^T – нижние треугольные матрицы над R, U и Q^TUQ – верхние треугольные матрицы над R, $D = \mathrm{diag}(d_1^{-1}, d_2^{-1}, .., d_r^{-1}, 0, .., 0)$ – диагональная матрица ранга r, $d_i \in R\backslash\{0\}$, $i = 1, \ldots, r$.

Key words: треугольное разложение матрицы; алгоритм в коммутативных областях.

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THE LAPLACE TRANSFORM METHOD FOR SOLVING DIFFERENTIAL EQUATIONS WITH DELAYED ARGUMENT

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Key words: Laplace transform method; differential equations with delayed argument.

The Laplace transform method is used for differential equations with delayed argument. There is constructed an algorithm, which is symbolic-numerical. The numerical component concerns a representation of functions, involved into the process by some kind of series.

There is a class of physical problems, which is associated with action of some kind of complementary forces - forces which are involved at various not initial time moments. Such problems frequently lead to the so called differential equations with delayed argument. Different ways of dealing with such equations exist. Applications of the Laplace transform method are well known. However there are some facts which prevent using this method in a symbolic way. Some difficulties, for example, are connected with a form of the solution of the Laplace image of the input differential equations. Only concrete kinds of equations may be solved entirely symbolically.

We restrict ourselves to the consideration of one equation, but the method works similarly with systems of equations of such type.

All functions of the argument t are supposed to satisfy the conditions for existing of their Laplace transform, and they equal zero for negative t. The points $t_k, t_{k-1} < t_k$, are taken in the set of $t, t \geqslant 0$. Consider an equation

$$x^{(n)}(t) + \sum_{j=1}^{n} \sum_{k=0}^{N} a_{jk} x^{(n-j)}(t - t_k) = f(t), \tag{1}$$

with initial conditions $x^{(n-j)}(0) = x_0^{(n-j}), j = 1, ldots, n$. The function $f(t)$ in the right-hand part is in general composite. We may consider for it the same partition points t_k. The unknown